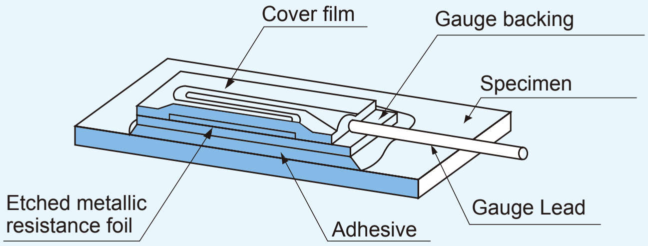
GENERAL DESCRIPTION

Strain gauges are generally used for one of three reasons:

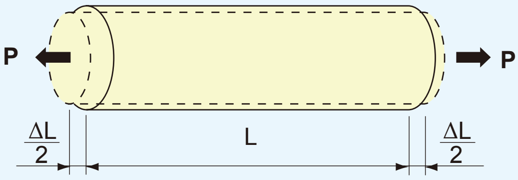
- To ascertain the amount of deformation caused by strain
- To ascertain the stress caused by strain and the degree of safety of a material or of a structural element that uses that material.
- To indirectly ascertain various physical quantities by converting them to strain.

There are a number of ways of measuring strain mechanically and electrically, but the vast majority of stress measurement is carried out using strain gauges due to their superior measurement characteristics.



What is Strain?

External force applied to an elastic material generates stress, which subsequently generates deformation in the material. At this time, the length of the material L extends to L+ΔL if the applied force is a tensile force. The ratio of ΔL to L, that is ΔL/L, is called strain. On the other hand, if a compressive force is applied, the length L is reduced to L−ΔL. Strain at this time is (−ΔL/L).



$$\varepsilon = \frac{\Delta L}{L}$$

where ε : Strain
L : Original length of material
ΔL : Change in length due to force P

Example) when a material of 100mm long deforms by 0.1mm in its length, the resulting strain is as follows.

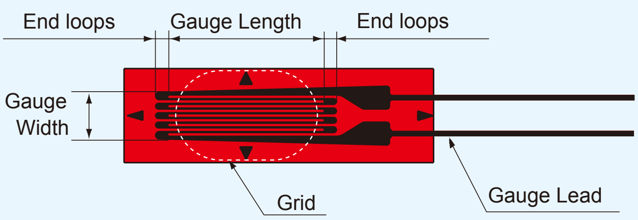
$$\varepsilon = \frac{\Delta L}{L} = \frac{0.1}{100} = 0.001 = 1000 \times 10^{-6}$$

What is a Strain Gauge?

The electric resistance of a metal changes proportionally to the mechanical deformation caused by an external force applied to the metal. By bonding a thin metal to a measurement object through a thin electrical insulator, the metal deforms depending on deformation of the measurement object and its electric resistance changes. The strain gauge (electric resistance strain gauge) is a sensor to measure the strain by means of measuring the resistance change.

Strain Gauge Configuration

A strain gauge is constructed by forming a grid made of fine electric resistance wire or photographically etched metallic resistance foil on an electrical insulation base (backing), and attaching gauge leads.



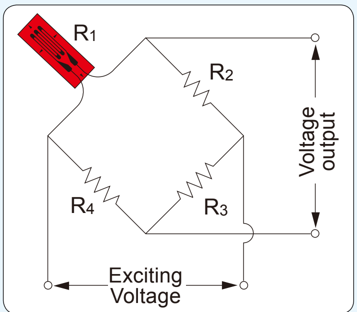
Strain Gauge Principles

When strain is generated in a measurement object, the strain is transferred to the resistance wire or foil of the strain gauge via the gauge base (backing). As a result, the wire or foil experiences a resistance change. This change is exactly proportional to the strain as in the equation below.

$$\varepsilon = \frac{\Delta L}{L} = \frac{\Delta R / R}{K}$$

where
ε : Strain measured
L : Original length of material
ΔL : Change in length due to force P
R : Gauge resistance
ΔR : Resistance change due to strain
K : Gauge Factor as shown on package

Normally, this resistance change is very small and requires a Wheatstone bridge circuit to convert the small resistance change to a more easily measured voltage change.



The voltage output of the circuit is given as follows.

$$e = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_2)(R_3 + R_4)} E$$

where
e : Voltage output
E : Exciting voltage
R_1 : Gauge resistance
R_2~R_4 : Fixed resistance

Here, if $R=R_1=R_2=R_3=R_4$ the resistance of the strain gauge changes to R+ΔR due to strain. Thus, the output voltage Δe (variation) due to the strain is given as follows.

$$e = \frac{\Delta R}{4R + 2\Delta R} E$$

When ΔR« R,

$$\Delta e = \frac{\Delta R}{4R} E = \frac{E}{4} K\varepsilon$$

When measuring with a strain gauge, it is connected to an instrument called a strainmeter. The strainmeter configures a Wheatstone bridge circuit and supplies exciting voltage. Measured strain is indicated on a digital display and/or output as analog signals.

Plane Stress and Strain

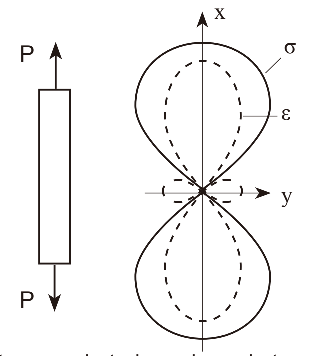
The stress in a material balanced with an applied external force can be considered a combination of more than one simple stress. In other words, these stresses can be divided into simple stress in the respective axial directions; however, measurement with ordinary strain gauges is restricted to the plane strain. In case that the stress exists in uniaxial direction like tension of a bar illustrated below, the following equation are applicable.

$$\varepsilon_x = \frac{\sigma}{E}$$

$$\varepsilon_y = -\nu\varepsilon_x = -\frac{\nu\sigma}{E}$$

where

- σ : Stress
- E : Elastic modulus
- ε_x : Strain in x direction
- ε_y : Strain in y direction
- ν : Poisson's ratio

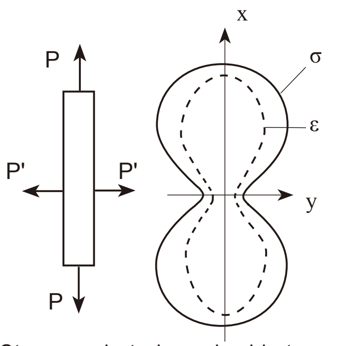


Stress and strain under uni-stress condition

The biaxial stresses generated by pulling the bar in both normal and transversal directions are:

$$\varepsilon_x = \varepsilon_x' - \nu\varepsilon_y' = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\varepsilon_y = \varepsilon_y' - \nu\varepsilon_x' = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$



Stress and strain under bi-stress condition

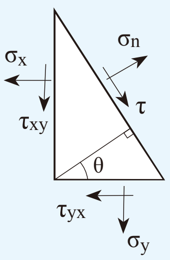
ε_x' : strain in the x direction due to σ_x

ε_y' : strain in the y direction due to σ_y

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

For the stress in other than the crossed biaxial directions, it is shown according to its angle as follows.



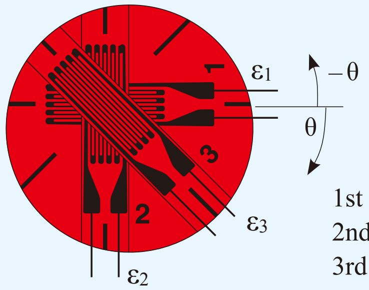
$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau_{xy}\sin 2\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos\theta$$

As noted from the above equations, in a certain direction, the maximum value of the resultant stress appears in the uniaxial diretion. The axial direction is called a principal direction of stress and the stress in that direction a principal stress. In this direction, the shearing stress is zero. The maximum value of shearing stress is generated in the direction of 45° against the principal direction of stress. It can also be applied to the strain. The strain in such a direction is called a principal strain.

Measurement of principal strain and stress using 3-element rectangular rosette gauge

When strain is generated in the surface of material and the principal direction of the strain and its extent are unknown, the principal strain, stress and their directions and shearing strain and stress can be obtained by measuring the strains in three directions over the surface. In order to simplify calculation, the relative angle in the three directions are determined as follows.



1st axis : ε_1
2nd axis : ε_2 at 90° position
3rd axis : ε_3 at 45° position

Maximum principal strain

$$\varepsilon_{max} = \frac{1}{2}\left[\varepsilon_1 + \varepsilon_2 + \sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Minimum principal strain

$$\varepsilon_{min} = \frac{1}{2}\left[\varepsilon_1 + \varepsilon_2 - \sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Maximum shearing strain

$$\gamma_{max} = \sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}$$

Angle from ε_1 gauge to direction of principal strain

$$\theta = \frac{1}{2}\tan^{-1}\left\{\frac{2\varepsilon_3 - (\varepsilon_1 + \varepsilon_2)}{\varepsilon_1 - \varepsilon_2}\right\}$$

If $\varepsilon_1 > \varepsilon_2$, the angle to the maximum principal strain is rotated by θ clockwise from the 1st axis, and the minimum principal strain is located at θ+90°. If $\varepsilon_1 < \varepsilon_2$, the angle to the maximum principal strain is rotated by θ+90° clockwise from the 1st axis, and the minimum principal strain is located at θ.

Maximum principal stress

$$\sigma_{max} = \frac{E}{1-\nu^2}(\varepsilon_{max} + \nu\varepsilon_{min}) = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_2}{1-\nu} + \frac{1}{1+\nu}\sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Minimum principal stress

$$\sigma_{min} = \frac{E}{1-\nu^2}(\varepsilon_{min} + \nu\varepsilon_{max}) = \frac{E}{2}\left[\frac{\varepsilon_1 + \varepsilon_2}{1-\nu} - \frac{1}{1+\nu}\sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}\right]$$

Maximum shearing stress

$$\tau_{max} = \frac{E}{2(1+\nu)}\gamma_{max} = \frac{E}{2(1+\nu)}\sqrt{2\{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\}}$$

NOTE:
The above rosette analysis equations are based on the 3-element strain gauge shown in the diagram. When the order of the axis numbers is different or when the gauge is not a 90° rosette gauge, different equations must be used. Check the axis numbers of applicable strain gauge before performing rosette analysis.